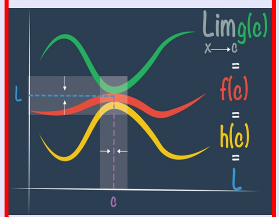
Calculus I

Lecture 16



Feb 19-8:47 AM

Class Quiz 16

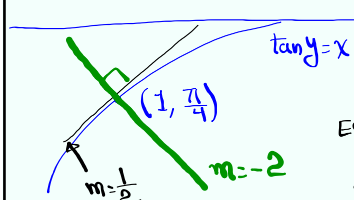
Given $\tan y = x$, Find $\left.\frac{dy}{dx}\right|_{(1, \frac{\pi}{4})}$.

$\frac{d}{dx}[\tan y] = \frac{d}{dx}[x]$

$\sec^2 y \cdot \frac{dy}{dx} = 1$ $\qquad \frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\frac{dy}{dx} = \cos^2 y$ $\qquad \left.\frac{dy}{dx}\right|_{(1, \frac{\pi}{4})} = \cos^2 \frac{\pi}{4} = \left[\frac{\sqrt{2}}{2}\right]^2$

$= \frac{2}{4} = \boxed{\frac{1}{2}}$ ✓



$\tan \frac{\pi}{4} = 1$ ✓

Eqn of tan. line

$y - \frac{\pi}{4} = \frac{1}{2}(x-1)$

$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$

Eqn of normal line

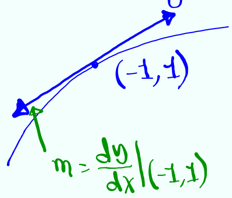
$y - \frac{\pi}{4} = -2(x-1)$

$\boxed{y = -2x + 2 + \frac{\pi}{4}}$

Apr 14-7:47 AM

Find eqn of the normal line to the graph of

$x^3 - 2xy^2 + y^3 = 2$ at $(-1,1)$.



$(-1)^3 - 2(-1)(1)^2 + 1^3 \stackrel{?}{=} 2$

$-1 + 2 + 1 \stackrel{?}{=} 2$

$2 = 2$ ✓

$$\frac{d}{dx}\left[x^3\right] - 2\frac{d}{dx}\left[xy^2\right] + \frac{d}{dx}\left[y^3\right] = \frac{d}{dx}\left[2\right]$$

$$3x^2 - 2\left[1\cdot y^2 + x\cdot 2y\cdot\frac{dy}{dx}\right] + 3y^2\cdot\frac{dy}{dx} = 0$$

$$3(-1)^2 - 2\left[1\cdot(1)^2 + (-1)\cdot 2\cdot(1)\cdot m\right] + 3(1)^2\cdot m = 0$$

$$3 - 2\left[1 - 2m\right] + 3m = 0$$

$$3 - 2 + 4m + 3m = 0$$

$$1 + 7m = 0$$

$\boxed{m = \frac{-1}{7}}$ slope of tan. line

$m_{\text{Normal line}} = 7$

$y - y_1 = m(x - x_1)$

$y - 1 = 7(x - -1)$

$\boxed{y = 7x + 8}$

Apr 16-9:07 AM

Use linear approximation to estimate $\frac{1}{\sqrt{4.1}}$.

$f(x) \approx f(a) + f'(a)(x-a)$

$\frac{1}{\sqrt{4.1}} \approx \frac{1}{\sqrt{4}} = \frac{1}{2}$

$f(x) = \frac{1}{\sqrt{x}}$ $\quad f(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$a = 4$

$f(x) = \frac{1}{x^{1/2}}$ $\quad f(x) = x^{-1/2}$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}} = \frac{-1}{2x^{3/2}} = \frac{-1}{2x\sqrt{x}}$$

$$f'(4) = \frac{-1}{2\cdot 4\sqrt{4}} = \frac{-1}{16}$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\frac{1}{\sqrt{x}} \approx \frac{1}{2} + \frac{-1}{16}(x-4)$$

$\frac{1}{\sqrt{4.1}} \approx .494$ by Calc.

$$\frac{1}{\sqrt{4.1}} \approx \frac{1}{2} - \frac{1}{16}(4.1-4) = \frac{1}{2} - \frac{1}{16}\cdot(.1)$$

$$= \frac{1}{2} - \frac{1}{160} = \boxed{\frac{79}{160}}$$

$\frac{79}{160} \approx \boxed{.494}$ Linear Approximation

Apr 16-9:18 AM

Use linear approximation to evaluate $\csc^2 31^\circ$.

$\csc^2 31^\circ = \dfrac{1}{\sin^2 31^\circ} \approx \dfrac{1}{\sin^2 30^\circ} = \dfrac{1}{\left[\frac{1}{2}\right]^2} = \dfrac{1}{\frac{1}{4}} = 4$

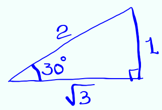
$f(x) = \csc^2 x$ $\qquad f(x) \approx f(a) + f'(a)(x-a)$

$a = 30^\circ$ $\qquad \csc^2 x \approx 4 - 8\sqrt{3}(x - 30^\circ)$

$f(30^\circ) = \csc^2 30^\circ = \left[\dfrac{1}{\sin 30^\circ}\right]^2 = 4$

$f'(x) = 2\csc x \cdot -\csc x \cot x = -2\csc^2 x \cot x$

$f'(30^\circ) = -2\csc^2 30^\circ \cdot \cot 30^\circ = -2 \cdot 4 \cdot \sqrt{3} = -8\sqrt{3}$



$\cot 30^\circ = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

$\csc^2 x \approx 4 - 8\sqrt{3}(x - 30^\circ)$ $\quad \frac{\pi}{6}$

Degree

Rad.

$\csc^2 31^\circ \approx 4 - 8\sqrt{3}(31^\circ - 30^\circ) = 4 - 8\sqrt{3} \cdot 1^\circ$

$= 4 - 8\sqrt{3} \cdot \dfrac{\pi}{180}$

Now by Calc.

$\csc^2 31^\circ = \dfrac{1}{\sin^2 31^\circ}$ $\qquad \approx \boxed{3.758}$ By linear Appr.

$\approx \boxed{3.770}$

Apr 16-9:29 AM

Suppose $V = V(t)$, and $r = r(t)$

Given $V = \frac{1}{3}\pi r^3$ $\quad$ find $\dfrac{dr}{dt}$

$\dfrac{d}{dt}[V] = \dfrac{d}{dt}\left[\boxed{\tfrac{1}{3}\pi} r^3\right]$

$\dfrac{dV}{dt} = \dfrac{\pi}{3} \dfrac{d}{dt}[r^3]$

$\dfrac{dV}{dt} = \dfrac{\pi}{\cancel{3}} \cdot \cancel{3} r^2 \cdot \dfrac{dr}{dt}$

$\dfrac{dV}{dt} = \pi r^2 \dfrac{dr}{dt}$

$\dfrac{\frac{dV}{dt}}{\pi r^2} = \dfrac{dr}{dt}$

$\boxed{\dfrac{dr}{dt} = \dfrac{1}{\pi r^2} \dfrac{dV}{dt}}$

Apr 16-9:47 AM

Suppose $V = V(t)$, $r = r(t)$, $h = h(t)$, $\boxed{V = \pi r^2 h}$

Find $\frac{dV}{dt}$ when $r = 2$, $h = 5$, $\frac{dr}{dt} = -2$, $\frac{dh}{dt} = 4$!

$$\frac{dV}{dt} = \frac{d}{dt}\left[\pi r^2 h\right]$$

$$\frac{dV}{dt} = \pi \frac{d}{dt}\left[r^2 h\right]$$

$$= \pi\left[2r \cdot \frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt}\right]$$

$$\left.\frac{dV}{dt}\right|_{\substack{r=2\\h=5}} = \pi\left[2 \cdot 2 \cdot (-2) \cdot 5 + 2^2 \cdot 4\right]$$

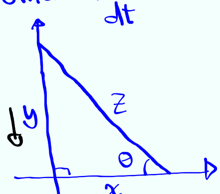
$$= \pi\left[-40 + 16\right] = \boxed{-24\pi}$$

Apr 16-9:52 AM

Suppose $x = x(t)$, $y = y(t)$, $\theta = \theta(t)$

Find $\frac{d\theta}{dt}$ when $x = 6$, $y = 8$,

$\frac{dx}{dt} = 3$, $\frac{dy}{dt} = -2$

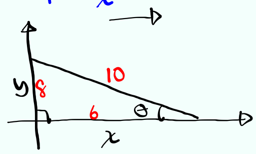


Right Triangle

$x^2 + y^2 = z^2$

when $x=6$, $y=8$

$6^2 + 8^2 = z^2 \rightarrow z = 10$



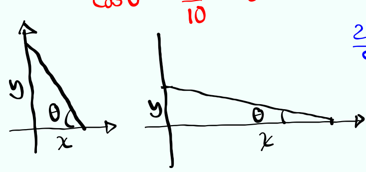
$\tan\theta = \frac{y}{x}$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{\frac{dy}{dt} \cdot x - y \cdot \frac{dx}{dt}}{x^2}$$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{6}{10}} = \frac{10}{6} = \frac{5}{3}$

$$\left(\frac{5}{3}\right)^2 \frac{d\theta}{dt} = \frac{-2 \cdot 6 - 8 \cdot 3}{6^2}$$

$$\frac{25}{9} \frac{d\theta}{dt} = \frac{-36}{36}$$



$$\frac{25}{9} \frac{d\theta}{dt} = -1$$

θ is getting Smaller

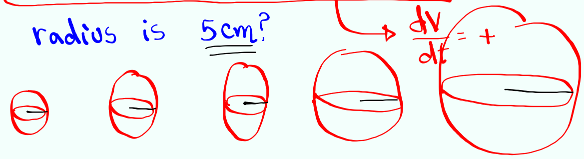
$$\frac{d\theta}{dt} = \frac{-9}{25}$$

Apr 16-9:58 AM

Sussie is putting air into a basketball.
The $\boxed{\text{radius}}$ of the ball is $\boxed{\text{increasing}}$ at the rate of $\boxed{2\text{cm/min.}}$ $\frac{dr}{dt} = 2\text{cm/min}$

$\boxed{\text{How fast its Volume changes}}$ when the radius is 5cm?

$\frac{dV}{dt} = +$



what is the Formula for volume of a ball?
Sphere

$$V = \frac{4\pi r^3}{3}$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$$

$$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$$

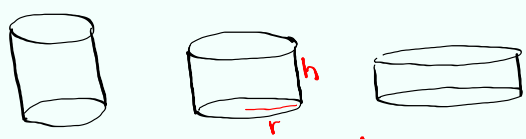
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi \cdot 5^2 \cdot 2$$

$$= 200\pi \ \text{cm}^3/\text{min}.$$

Apr 16-10:12 AM

A right circular cylinder is being impacted such that its height decreases at 4 in./min and its radius increases at 6 in./min.



$$\frac{dh}{dt} = -4, \quad \frac{dr}{dt} = 6$$

how fast its volume changes when $h = 2$ in. and $r = 10$ in.?

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right]$$

$$= \pi\left[2 \cdot 10 \cdot 6 \cdot 2 + 10^2 \cdot (-4)\right]$$

$$= \pi[240 - 400] = -160\pi \ \text{in}^3/\text{min}.$$

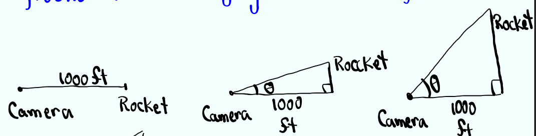
Volume is decreasing.

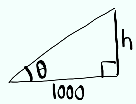
Apr 16-10:26 AM

A camera is on the ground 1000 ft from the launching pad of a rocket.

Rocket goes up vertically with speed of 2500 ft/min. $\frac{dh}{dt} = 2500$ ft/min.

How fast is the angle between the camera and ground level changing when the angle is $\boxed{30°}$?





$\tan\theta = \frac{h}{1000}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{1000}\frac{dh}{dt}$

$\sec^2 30° \frac{d\theta}{dt} = \frac{1}{1000} \cdot 2500$

$\frac{d\theta}{dt} = \frac{2.5}{\sec^2 30°} = 2.5\cos^2 30°$

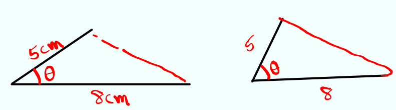
$= 2.5\left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{7.5}{4} = \boxed{1.875 \text{ Rad/min.}}$

Apr 16-10:39 AM

A triangle has two sides of 5cm and 8cm. The angle between them is increasing at the rate of $\boxed{1 \text{ rad/min.}}$ $\frac{d\theta}{dt} = 1$

How fast its area changing when the angle is $\boxed{60°?}$



Area of a triangle.

$\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$

$A = \frac{1}{2} \cdot 5 \cdot 8 \cdot \sin\theta$

$A = 20\sin\theta$

$\frac{dA}{dt} = 20 \cdot \cos\theta \cdot \frac{d\theta}{dt}$

$= 20 \cdot \cos 60° \cdot 1$

$= 20 \cdot \frac{1}{2} \cdot 1$

$\boxed{\frac{dA}{dt} = 10 \text{ cm}^2/\text{min.}}$

Apr 16-10:52 AM

$f(x) = x^2 + 8x + 4$ $\quad f(-4) = (-4)^2 + 8(-4) + 4$

$= 16 - 32 + 4$

$= -12$

1) $f'(x) = 2x + 8$

2) Solve $f'(x) = 0$ $\quad 2x + 8 = 0 \quad x = -4$

3) Find Points on the graph of $f(x)$ where $f'(x) = 0$. $\quad (-4, f(-4)) = (-4, -12)$

4) Find $f''(x)$ $\quad f''(x) = 2$

5) Solve $f''(x) = 0$ $\quad 2 = 0$ $\quad$ No solution

6) Find points on the graph of $f(x)$ where $f''(x) = 0$. $\quad$ NO Such points

Apr 16-11:03 AM